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Modeling the Curved Turbulent Wall Jet

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The paper deals with the application of a Reynolds stress closure model of turbulence to the calculation of wall jets developing on plane and convex curved surfaces. Good results are obtained for the mean flowfield, the growth rate of the jet, and, in particular, the distance between the points of zero shear stress and zero mean-velocity gradient, which increases as the result of the suppression by convex curvature of the turbulence in the inner layer. The turbulence field is less well predicted, although the results are not significantly inferior to those obtained previously for curved-wall boundary layers. Deficiencies are identified in the modeling of turbulent diffusion and energy dissipation rate.

Nomenclature

b	= jet slot height (Fig. 1)
c_f	= skin-friction coefficient defined with respect to $\frac{1}{2}\rho U_J^2$
F	= factor applied to mixing length for curved flow
n	= distance measured normal to curved surface
$n_{1/2}$	= normal distance to half-velocity point (Fig. 1)
P	= production rate of turbulent kinetic energy
P_{ij}	= production rate of Reynolds stress $\overline{u_i u_j}$
p	= fluctuating pressure
$\overline{q^2}$	= 2 × turbulent kinetic energy ($=\overline{u^2}+\overline{v^2}+\overline{w^2}$)
R	= surface radius of curvature
r	= local streamline radius of curvature ($\simeq R+n$)
s	= distance measured along curved surface
U	= streamwise component of mean velocity
U_∞	= velocity at edge of wall jet (plane flow only)
U_m	= maximum mean velocity
U_0	= $U_m - U_\infty$
U_J	= velocity in slot exit plane
u,v,w	= components of fluctuating velocity in s,n,z directions
α,β,γ	= interdependent constants in pressure-strain model, Eq. (2)
β	= empirical constant in mixing-length modification, Eq. (1)
δ	= distance from surface of velocity maximum
δ'	= distance from surface to point of zero shear stress
ϵ	= dissipation rate of turbulent energy

Introduction

THE turbulent wall jet is an interesting flow because it consists of a wall layer and a free shear layer interacting with each other, and thus it possesses many of the characteristics of both. It is also an example of a relatively simple shear layer in which the point of turbulent shear stress reversal does not coincide with the point of maximum mean velocity but lies inside it, closer to the wall. This feature of the flow obviously cannot be predicted by simple gradient transport models of turbulence, although it turns out that, for the wall jet developing on a flat surface, the zeros of shear stress and mean-velocity gradient are close enough to one another for the difference to be overlooked, so that eddy viscosity models can be used to describe the principal features of the flow.[1] In the case of a wall jet developing on a longitudinally curved surface, the turbulence structure is affected by the curvature in ways that cannot be accounted for satisfactorily in terms of simple gradient hypotheses. If, for example, the surface curvature is convex, the extra strain introduced by it tends to reduce the turbulence intensity and shear stress in the near-wall layer, where the angular momentum of the flow increases in the direction of the radius of curvature. The reverse occurs in the jet-like layer outside the velocity maximum, where turbulent activity is increased by the curvature. Curved-wall boundary layers and other curved shear layers without a velocity maximum may be calculated with fair accuracy when the ordinary plane flow mixing-length distribution is modified, as suggested by the buoyancy analogy,[2,3] by a factor

$$F = 1 - \beta \frac{U/r}{\partial U/\partial n} \equiv 1 - \beta S \qquad (1)$$

Unfortunately, it turns out that β is not universally constant for different types of curved flow, and this lack of generality has led to abandonment of Eq. (1) in favor of predictive schemes based on the numerical solution of modeled transport equations for the Reynolds stresses that, in the case of curved flow, contain in exact form extra-strain production terms introduced by the curvature.[4,5] The shortcomings of eddy viscosity modeling are yet further emphasized by the curved-wall jet. Intuition suggests, and experiment confirms, that the suppression of turbulence in the wall layer and its augmentation in the outer flow result in an inward shift of the zero shear stress point relative to the velocity maximum. The distance between these two points is too great to be ignored, turbulent stress diffusion assumes an important role, and local conditions depart from the quasilocal equilibrium implicitly assumed by the simpler models. It appears that the minimum requirement for a satisfactory prediction method in this case is that a transport equation for the shear stress be solved. The importance of turbulent transport makes this flow an important and stringent test of transport-equation modeling.

The flow in wall jets is well documented. In recent years a number of detailed turbulence measurements have been published. The literature has recently been critically reviewed by Launder and Rodi[6] for the 1980-81 AFOSR-Stanford Conference on calculation of complex turbulent shear flows. Of four studies of constant-radius, convex, curved flow, the most recent, and by a considerable margin the most detailed, is that of Alcaraz,[7,8] in which a relatively low slot-height-to-radius ratio ($b/R = 0.0032$) was used to obtain a closely two-dimensional flow. The curvature was, however, large enough to produce measurable effects on the turbulence structure along the lines previously described, with $n_{1/2}/R$ reaching a maximum value of 0.025. A self-preserving form is achieved

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when the wall jet develops on a logarithmic spiral; of three sets of data reviewed by Launder and Rodi,[6] the most closely two-dimensional was that of Guitton and Newman,[9] which is also particularly suitable for stress model validation in that hot-wire data are provided for all four nonzero Reynolds stresses. The curvature is an order of magnitude greater than in Alcaraz's experiment, with $n_{½}/R$ equal to 0.27 in the self-preserving flow.

Proposals for closing the Reynolds stress equations abound in the literature and form the bases for numerous prediction methods that have been applied to complex shear layers with varying degrees of success. Irwin and Arnot Smith[10] first used the method to calculate curved flow with the model of Launder et al.[11] The treatment was formally limited to shear layers with mild longitudinal curvature by the use of the plane boundary-layer form of the mean momentum equation and consequent neglect of the cross-stream variation of pressure. Results were obtained for curved-wall jets, boundary layers, and a curved free jet and demonstrated the capability of Reynolds stress modeling to predict the influence of curvature on flow development, although on account of the limitation noted the agreement with experiment deteriorated for flows of large curvature, the rate of growth of the wall jet[9] being significantly underpredicted. Moreover, the limited amount of turbulence (as distinct from mean field) data available at the time of this pioneering study meant that searching comparisons could not be undertaken.

Irwin and Arnot Smith[10] modeled the difficult pressure-strain redistribution terms in the Reynolds stress equations in the way suggested by Launder et al.[11]:

$$\overline{\frac{p}{\rho}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)} = -C_1\frac{\epsilon}{\overline{q^2}}\left(\overline{u_iu_j}-\frac{1}{3}\delta_{ij}\overline{q^2}\right)$$

(a)

$$-\alpha\left(P_{ij}-\frac{2}{3}\delta_{ij}P\right)-\beta\overline{q^2}\left(\frac{\partial U_i}{\partial x_j}+\frac{\partial U_j}{\partial x_i}\right)$$

(b) (c)

$$-\gamma\left(D_{ij}-\frac{2}{3}\delta_{ij}P\right) \tag{2}$$

(d)

where:

$$P_{ij}=-\overline{u_iu_k}\frac{\partial U_j}{\partial x_k}-\overline{u_ju_k}\frac{\partial U_i}{\partial x_k} \tag{3}$$

$$D_{ij}=-\overline{u_iu_k}\frac{\partial U_k}{\partial x_j}-\overline{u_ju_k}\frac{\partial U_k}{\partial x_i} \tag{4}$$

α, β, and γ are single functions of a single constant evaluated with C_1 from simple shear flow data. Term (a) in Eq. (2), originally due to Rotta,[12] represents the wholly turbulent contribution to pressure strain, and the remaining three terms are the mean-strain, or rapid, terms that Chou's[13] analysis showed should be present. Equation (2) requires some modification to take account of the influence of the wall on intercomponent energy transfer; these effects were modeled differently in Refs. 10 and 11, but the details are not essential to the present discussion.

Recently, the present authors[5] used an abbreviated form of Eq. (2), in which only terms (a) and (b) appeared, to calculate four curved-wall boundary layers with fairly satisfactory results. The isolation of the "anisotropy of production" term (b), which Launder et al.[11] recognized in their original paper as the dominant one in the rapid part of pressure strain, permits different interpretations of P_{ij}. When the Reynolds stress equations are transformed from Cartesian to curvilinear coordinates, there appear extra generation terms due to rotation of axes. Although these have the same form as the other stress-generation terms—the product of a Reynolds stress and a mean rate of strain (in this case the "extra" strain U/r)—they do not appear in P_{ij} defined by Eq. (3) when this is transformed to curvilinear coordinates. Thus, in their study of swirling jet flow, Launder and Morse[16] used Eq. (2) and treated the equivalent extra terms as convection terms. In using the shortened form of Eq. (2) the present authors had no inhibitions about interpreting P_{ij} in term (b) as being the total production rate of $\overline{u_iu_j}$ including terms arising from axis rotation. The effects of model predictions for curved flow are significant. In the local-equilibrium limit, when mean flow and turbulent transport become negligible, the Reynolds stress equations reduce to a set of algebraic equations for the stress ratios.[14] Irwin and Arnot Smith showed that for these conditions the pressure-strain model equation (2) predicts collapse of the shear stress in curved flow when the curvature parameter S[defined in Eq. (1)] reaches a critical value of 0.1 in free flow and 0.085 when a near-wall correction factor is applied. In contrast our simpler model, which includes the axis rotation terms and constants evaluated from plane flow data, predicts[14] collapse at $S=0.17$. The measurements of So and Mellor[15] in a boundary layer subjected to severe stabilizing curvature suggested a critical value of about 0.15.

The present contribution may be regarded as an extension of our previous curved-wall boundary-layer calculations[5] to the rather more difficult case of a wall jet on a convex surface. It is unnecessary to repeat here the full description of the turbulence model and calculation method, but it may be helpful to itemize the principal features as follows:

1) The pressure-strain correlation of the Reynolds stress equations is modeled by an abbreviated form of Eq. (1):

$$\overline{\frac{p}{\rho}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)} = -C_1\frac{\epsilon}{\overline{q^2}}\left(\overline{u_iu_j}-\frac{1}{3}\delta_{ij}\overline{q^2}\right)$$
$$-C_2\left(P_{ij}-\frac{2}{3}\delta_{ij}P\right) \tag{5}$$

where P_{ij} is interpreted as discussed previously and C_1 and C_2 are constants, equal to 3.6 and 0.6, respectively, evaluated from plane flow data.

2) Both terms in Eq. (5) are modified by a wall damping factor based on an original proposal by Shir[17] and extended by Gibson and Launder[18] to account for wall effects on density-stratified turbulence.

3) Turbulent transport of Reynolds stress is modeled by a simple gradient diffusion hypothesis for the triple correlations:

$$\overline{u_iu_ju_k}=-C_t(\overline{q^2}/\epsilon)\overline{u_ku_l}(\partial\overline{u_iu_j}/\partial x_l) \tag{6}$$

4) The turbulence energy dissipation rate ϵ is obtained from a modeled transport equation identical in form to that proposed by Launder et al.[11]

5) The cross-stream variation of pressure in curved flow is accounted for in a marching boundary-layer procedure by assuming

$$\frac{1}{\rho}\frac{\partial p}{\partial n}=\frac{U^2}{r} \tag{7}$$

The practice adopted previously[5] was to use the measured surface pressure as the boundary condition and integrate through the boundary layer to the external potential flow. The two curved-wall jets considered in the present paper develop in stagnant surroundings, and the reverse-procedure was used to obtain the wall pressure by integration from uniform pressure conditions at the edge to the wall. The results agree fairly well with the few scattered data points given by Guitton and Newman[9]; measured wall pressures were not reported by Alcaraz.[8]

6) Near the wall the finite difference solution is matched to the logarithmic law of the wall consistent with the observed persistence of a logarithmic layer in curved flow albeit much diminished in extent by strong stabilizing curvature.

We have also tried the long expression Eq. (2) for pressure strain and various alternatives to the triple-correlation model Eq. (6), whose shortcomings are exposed by the wall jet, in which turbulent diffusion is more important than in a boundary layer.

In adapting Eq. (2) to wall flow we have used the Shir[17] correction of our other model, so that the full wall-flow version differs in detail from those of Irwin and Arnot Smith[10] and Launder et al.[11] The additional constants involved[5] are set equal to 0.5 and 0.1; these give approximately the correct stress levels for plane wall boundary-layer flow. Wall jet predictions are compared with the very detailed measurements of Alcaraz[7,8] and those of the self-preserving flow studied by Guitton and Newman[9]; the data of Irwin[19] are used to check model performance for plane wall jet flow.

Results and Discussion

We first checked the model and calculation method for plane flow using as a standard the self-preserving wall jet data published by Irwin.[19] The jet develops in a moving stream, and self-preservation is accomplished in a tailored pressure gradient; the maximum to freestream velocity ratio is 2.65. Figure 2 shows the measured and predicted development of the mean flow downstream of the injection slot. The symbols are defined in Fig. 1 with the exception of δ', which is the distance from the surface of the point of zero shear stress. The ratio δ/δ', in which δ is the distance from the surface of the mean-velocity maximum, is about 1.4 and increases very slightly with distance measured downstream, a slight departure from exact self-preservation. This value and the slight increase are closely reproduced by the model, as are the growth rate, decay of maximum velocity, and streamwise variation of skin-friction displayed in Fig. 2. Profiles of mean velocity and of the four nonzero components of Reynolds stress are shown in Fig. 3. Two sets of predictions are represented by continuous and broken lines; the latter were obtained from the Irwin-Smith pressure-strain hypothesis, Eq. (2), the former from Eq. (5). The differences here, for plane flow, are within the data scatter; both models tend to

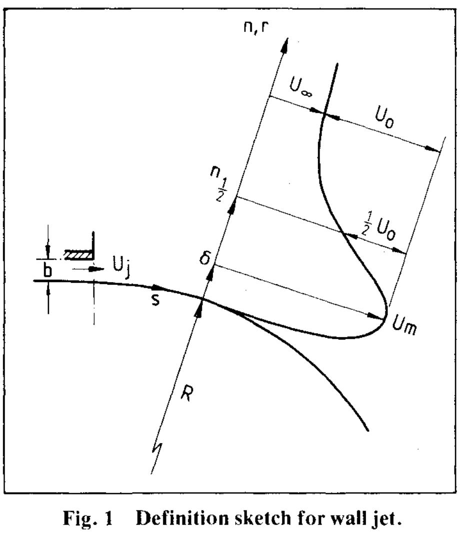

Fig. 1 Definition sketch for wall jet.

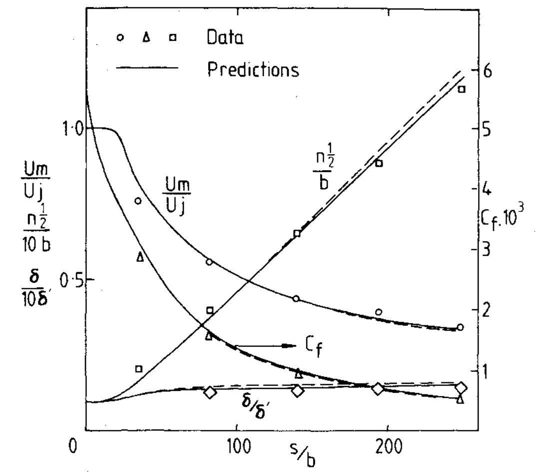

Fig. 2 Measured and predicted development of a plane self-preserving wall jet. Data of Irwin.[19] Predictions: —— Eq. (5); --- Eq. (2).

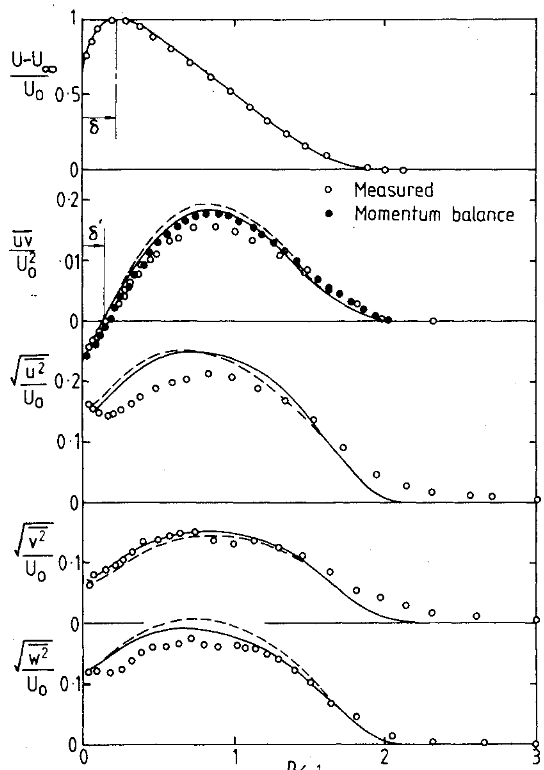

Fig. 3 Profiles of mean velocity and Reynolds stresses in a plane self-preserving wall jet. Data of Irwin.[19] Predictions: —— Eq. (5); --- Eq. (2).

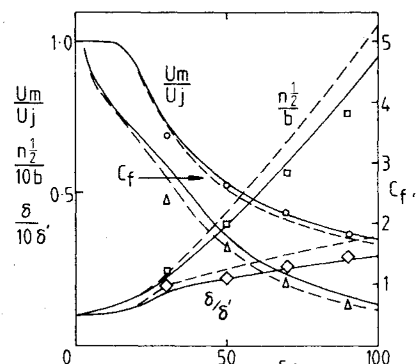

Fig. 4 Measured and predicted development of a wall jet on a convex surface. Data of Alcaraz.[7,8] $b/R=0.0032$. Predictions: —— Eq. (5); --- Eq. (2).

overpredict the turbulent kinetic energy just outboard of the velocity maximum, mainly because high values of $\overline{u^2}$ are calculated. Otherwise the agreement appears to be very good, apart perhaps from the slight discrepancies observed at the edge of the jet. We expected good mean-field predictions because Ljuboja and Rodi[1] had demonstrated that these were obtainable with the equivalent algebraic stress model (which reduces to an eddy viscosity form). It was pleasing to find that the full stress model reproduced so closely the distance between the points of zero stress and maximum velocity, which must have been calculated as coincident in Ref. 1.

Turning now to the curved-wall jet, Fig. 4 shows the predicted development of the mean flow compared with the measurements of Alcaraz.[7,8] The agreement, although satisfactory, is not quite as good as for the plane jet. Significant differences appear in the results given by the two pressure-strain hypotheses, particularly in respect to the ratio δ/δ', which increases sharply and continues to increase in the curved flow to double the plane-layer value. This increase, which, as noted, is due to suppression by curvature of the turbulence in the near-wall layer and the reverse in the outer flow, is closely predicted by the model, whereas that of Irwin and Smith[2] tends, relatively, to exaggerate these effects. The predicted cross-stream variation of mean velocity and the Reynolds stresses plotted in Fig. 5 show reasonably good agreement with the data. Also plotted is the shear stress profile deduced from the mean-flow measurements and momentum equation; this shows some evidence of three-dimensionality in the destabilized outer layer, which possibly accounts for the deteriorating level of agreement there. As expected, the Irwin-Smith pressure-strain model (broken-line predictions) again predicts higher stress levels in the outer layer; in the inner flow the difference between the two models is less apparent.

Both models give fairly good results for the shear stress and $\overline{v^2}$ in the near-wall layer but neither reproduces the redistribution of energy between the other two components and, most noticeably, the significant increase in $\overline{w^2}$ to a value greater than that of $\overline{u^2}$ as the wall is approached. This phenomenon appears to be genuine, since it appears also in the measurements of Guitton and Newman[9] presented in Fig. 9.

The turbulence energy and shear stress balances presented by Alcaraz[7,8] provide a welcome opportunity to test the model assumptions in detail. The most noticeable feature of the measured energy balance shown in Fig. 6 is the important role apparently played by pressure diffusion in the wall layer. This rises to a sharp peak inboard of the velocity maximum, where it is offset to a large extent by velocity (triple-correlation) transport of the opposite sign. The net turbulent transport is thus relatively small but by no means insignificant: it and the energy dissipation rate appear to be the dominant terms in this region. The pressure diffusion data reproduced in Fig. 6 were obtained not by direct measurement but by difference to complete the energy balance. The net turbulent transport near the wall is quite well represented by the gradient diffusion hypothesis (6), although the level of agreement deteriorates further from the wall. Equation (6) has been criticized because it is not compatible in its symmetry properties, since only the left-hand side is independent of the order of the indices i, j, and k. Lumley[20] has suggested that a possible reason why Eq. (6) usually gives good results for wall flow may be that it absorbs some pressure diffusion that is not symmetrical and is not modeled explicitly. The present results tend to support this view, but we would not press the point too hard. Of the remaining terms in the energy balance, the mean-flow transport and production rate, which present no problems in measurement or prediction, are reasonably well predicted. The discrepancies noted for turbulent transport are reflected by the significant difference between measured and predicted values of the dissipation rate outside the velocity maximum. This apparent failure to reproduce the dissipation rate is rather surprising in view of good results obtained for $\overline{q^2}$ in this region and may be contrasted with the much better level of agreement obtained in this respect with the same models for a curved (stable) mixing layer.[4] The flow in the outer layer

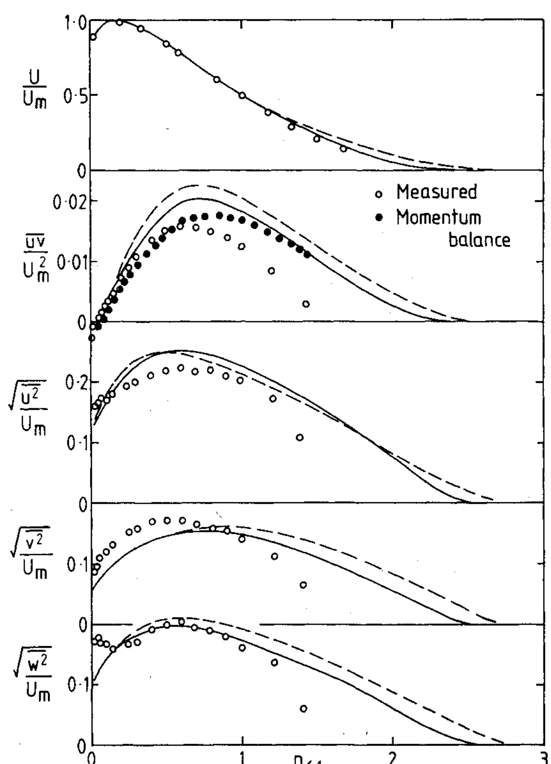

Fig. 5 Profiles of mean velocity and Reynolds stresses in a curved-wall jet. Data of Alcaraz.[7,8] Predictions: —— Eq. (5); --- Eq. (2).

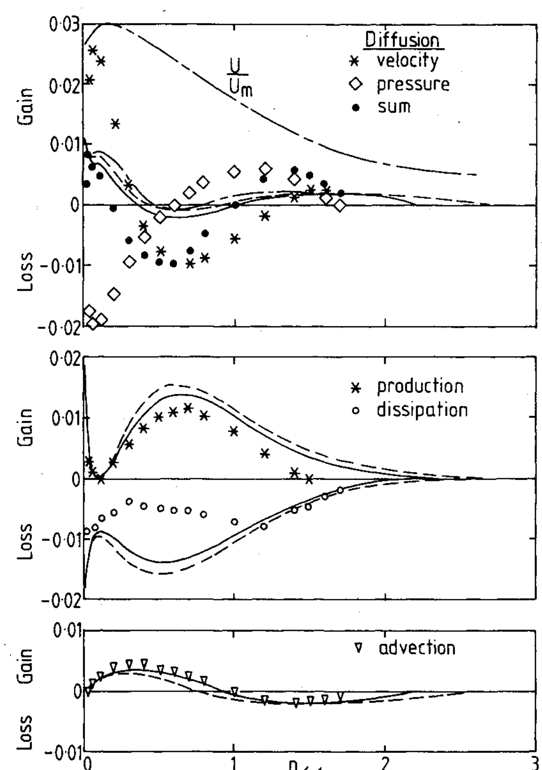

Fig. 6 Turbulent energy balance in a curved-wall jet. Data of Alcaraz.[7,8] Predictions: —— Eq. (5); --- Eq. (2), — • — diffusion Eq. (10).

is not unlike that in a mixing layer, but these results seem to be at variance with those previously reported for such flows (see, for example, the review by Rodi[21]) and with what might be expected intuitively. The dissipation rate was obtained by Alcaraz from the isotropic relation:

$$\epsilon = 15\nu \overline{\left(\frac{\partial u}{\partial x}\right)^2} \tag{8}$$

Closure of the modeled equations is effected by solving the simple and widely used equation recommended by Launder et al.[11] for the dissipation rate:

$$\frac{D\epsilon}{Dt} = C_\epsilon \frac{\partial}{\partial n}\left(\frac{r}{R}\frac{\overline{q^2}}{\epsilon}\overline{v^2}\frac{\partial \epsilon}{\partial n}\right) + \frac{\epsilon^2}{\overline{q^2}}\left(C_{\epsilon l}\frac{P}{\epsilon} - C_{\epsilon 2}\right) \tag{9}$$

This equation has given good results for simple shear layers, but its usefulness in more complex flows has recently been questioned. For example Launder and Morse[16] identified the modeling of the source terms in Eq. (9) as a major weakness in their Reynolds stress model of a swirling jet, and Rodi[22] described empirical modifications that improve performance for the analogous effects of buoyancy and rotation. Hanjalic and Launder[23] recommended inclusion in Eq. (9) of additional normal stress-generation terms. We found that neither of these proposals made significant differences in the predictions[4] of a curved mixing layer, and so, for the time being, we are content to retain Eq. (9) for curved-flow calculations.

The shear stress balance, Fig. 7, reveals high measured rates of turbulent transport in the vicinity of the velocity maximum which are not reproduced adequately by the simple gradient transport model Eq. (6), where the deficiency is offset in the predictions by an equivalent discrepancy in pressure strain, obtained from the measurements by difference. We have experimented with alternative models for the triple correlation without achieving significantly better results. Figures 6 and 7 show turbulent diffusion calculated from the invariant relationship recommended by Launder et al.[11]:

$$\overline{u_i u_j u_k} = -C_S \frac{\overline{q^2}}{\epsilon}\left[\overline{u_i u_l}\frac{\partial \overline{u_j u_k}}{\partial x_l} + \overline{u_j u_l}\frac{\partial \overline{u_k u_i}}{\partial x_l} + \overline{u_k u_l}\frac{\partial \overline{u_i u_j}}{\partial x_l}\right] \tag{10}$$

The result is to increase the net diffusion near the wall, but the change is very small.

The final comparison is made with the data of Guitton and Newman,[9] whose nominally self-preserving wall jet developed on a logarithmic spiral. The results for $s/R = 1$ presented in Figs. 8 and 9 show much the same level of agreement as that noted for the other flows. The ratio δ/δ' cannot be obtained accurately from the published data; the large predicted value of 7.0 appears to be broadly consistent with the measurements.

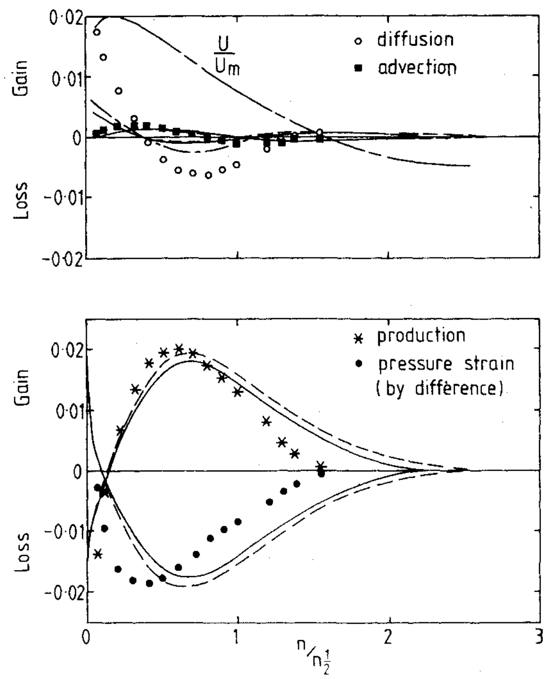

Fig. 7 Shear stress balance in a curved-wall jet. Data of Alcaraz.[7,8] Predictions: —— Eq. (5); --- Eq. (2), — • — diffusion Eq. (10).

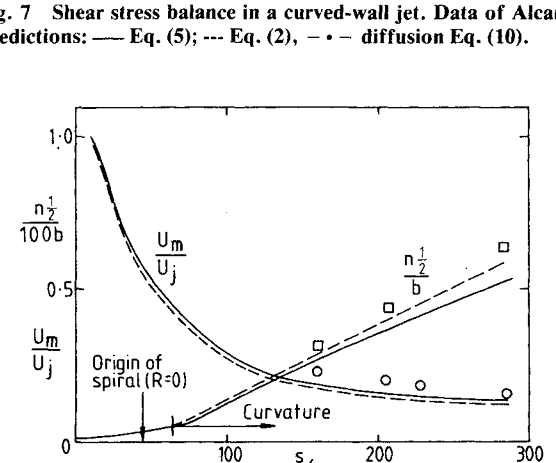

Fig. 8 Measured and predicted development of a wall jet on a logarithmic spiral. Data of Guitton and Newman.[9] Predictions: —— Eq. (5); --- Eq. (2).

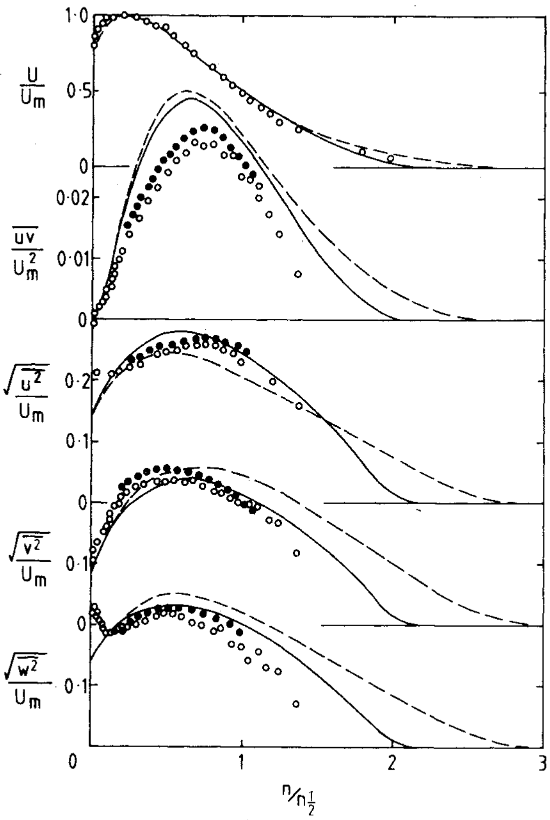

Fig. 9 Profiles of mean velocity and Reynolds stresses in a wall jet on a logarithmic spiral. Data of Guitton and Newman.[9] Predictions: —— Eq. (5); --- Eq. (2).

Concluding Remarks

The calculations presented in this paper have been carried out as part of an ongoing program of research motivated by the requirement for a generally applicable predictive procedure for complex shear flows with density stratification, rotation, and/or streamline curvature. The work is thus a continuation of our studies of buoyancy,[18] curved free flow,[4] and curved-wall flow[5] with the same turbulence model, and it is a logical extension of the latter. Moreover, the wall jet possesses interesting and individual features that are exaggerated when the flow develops on a convex curved surface. One of these, which can be predicted only with a stress equation model, is that the zeros of shear stress and mean-velocity gradient occur at different places. The present model accurately predicts the distance between these two points and the extent by which this distance is increased by the stabilizing action of convex surface curvature.

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